Marginal Fermi liquid behavior from 2d Coulomb interaction

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(February 1, 2008)

A full, nonperturbative renormalization group analysis of interacting electrons in a graphite layer is performed, in order to investigate the deviations from Fermi liquid theory that have been observed in the experimental measures of a linear quasiparticle decay rate in graphite. The electrons are coupled through Coulomb interactions, which remain unscreened due to the semimetallic character of the layer. We show that the model flows towards the noninteracting fixed-point for the whole range of couplings, with logarithmic corrections which signal the marginal character of the interaction separating Fermi liquid and non-Fermi liquid regimes.

71.27.+a, 73.20.Dx, 05.30.Fk

During recent years there has been important progress in understanding the properties of quantum electron liquids in dimension $D < 3$. One of the most fruitful approaches in this respect springs from the use of renormalization group (RG) methods, in which the different liquids are characterized by several fixed-points controlling the low-energy properties. The Landau theory of the Fermi liquid in dimension $D > 1$ can be taken as a paradigm of the success of this program. It has been shown that, at least in the continuum limit, a system with isotropic Fermi surface and regular interactions is susceptible of developing a fixed-point in which the interaction remains stable in the infrared[1].

The question of whether different critical points may arise at dimension $D = 2$ is now a subject of debate[2–5]. From the perspective of the RG approach, one of the premises leading to the Fermi liquid fixed-point should be relaxed in order to flow to a different universality class. In the case of models proposed to understand the electronic properties of copper-oxide superconductors, the high anisotropy of the Fermi surface[6] may play an important role in the anomalous behavior of the normal as well as of the superconducting state[7]. On the other hand, a possible source of non-Fermi liquid behavior may arise in systems with singular interactions[8]. In the case of the Coulomb interaction screened by the Fermi sea, a solution by means of bosonization methods has shown that no departures from Fermi liquid behavior arise at $D = 2$ and 3[9]. It has been also shown for the conventional screened interaction that only potentials as singular as $V(q) \sim 1/|q|^{2D-2}$ can lead to a different electron liquid[8,10]. The system of electrons with magnetic interactions is quite different in that respect. It is known that in this case the system shows non-Fermi liquid behavior for $D \leq 3$, manifested in properties like the specific heat or anomalous electron field dimensions[11–13].

In the present work, we address the applicability of the notion of Fermi liquid fixed-point to two-dimensional systems with unscreened Coulomb interaction. The absence of screening requires the vanishing of the density of states at the Fermi level. Semimetals show this behavior, while retaining a gapless electronic spectrum. The existence of a well defined continuum at low energies permits the existence of non trivial scaling properties[14]. The most remarkable example of this kind is given by the two-dimensional sheet of graphite, which has a vanishing density of states at the Fermi level[15]. Recent photoemission experiments in graphite, at intermediate energies, show a decay rate of quasiparticles proportional to their energy[16]. This represents a clear deviation with respect to the behavior in metals, which follow the conventional Fermi liquid picture with quasiparticle lifetimes propoportional to the inverse of the energy square, with, at most, logarithmic corrections. A description in terms of an effective field theory model has shown that the electronic interactions within the graphite layers are mainly responsible for the anomalous properties measured in the experiment[17].

We apply RG techniques to investigate whether the mentioned anomalous behavior can be understood as a marginal deviation from Fermi liquid theory, or rather it points towards a different universality class realized in the graphite sheet. We recall that the low-energy electronic excitations of the latter at half-filling are concentrated around two Fermi points at the corners of the hexagonal Brillouin Zone, where the dispersion relation is well approximated by two cones in contact at the apex. The effective field theory is given therefore by a pair of Dirac fermions, with a Coulomb potential that remains unscreened due to the vanishing density of states at the Fermi points. The effective hamiltonian can be written in the form[18]

$$H = -iv_F \int d^2r \Psi^+(\mathbf{r}) \boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \Psi(\mathbf{r}) + \frac{e^2}{8\pi} \int d^2r_1 \int d^2r_2 \Psi^+(\mathbf{r}_1)\Psi(\mathbf{r}_1) \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|} \Psi^+(\mathbf{r}_2)\Psi(\mathbf{r}_2) \qquad (1)$$

where $\Psi(\mathbf{r})$ is a two-dimensional Dirac spinor and $\boldsymbol{\sigma} \equiv (\sigma_x, \sigma_y)$. Such effective field theory provides a good starting point for a RG analysis since, given that the scaling dimension of the $\Psi(\mathbf{r})$ field is -1 (in length units), the four-fermion Coulomb interaction turns out to be scale invariant, at this level, with a dimensionless coupling constant e^2.

In order to address the existence of a new universality class, besides the trivial noninteracting phase, a nonperturbative approach has to be adopted, since such new phase can only be revealed by a nontrivial fixed-point in coupling constant space. In the following, we will implement a GW approximation in the computation of the self-energy properties. This is more easily achieved in the present model by replacing the four-fermion term in (1) by the interaction with an auxiliary scalar field used to propagate the Coulomb interaction. The effective hamiltonian can be rewritten in the form

$$H = -iv_F \int d^2r \Psi^+(\mathbf{r}) \boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \Psi(\mathbf{r}) + e \int d^2r \Psi^+(\mathbf{r}) \Psi(\mathbf{r})\; \phi(\mathbf{r}) \tag{2}$$

where the scalar field $\phi(\mathbf{r})$ has the propagator

$$i\langle T\phi(\mathbf{r},t)\; \phi(\mathbf{r}',t')\rangle = \frac{1}{4\pi}\delta(t-t')\frac{1}{|\mathbf{r}-\mathbf{r}'|} \tag{3}$$

In this framework we will introduce the GW approximation by taking into account the quantum corrections to the ϕ propagator due to particle-hole excitations of the Fermi sea. This kind of approximation has proven to be adequate to the description of the crossover from Fermi liquid to Luttinger liquid behavior upon lowering the dimension from $D = 2$ to 1, capturing the relevant physical processes in the electron system[19]. Therefore, it seems also appropriate to uncover any possible fixed-point, different from that of Fermi liquid theory, in the case of the system with unscreened Coulomb interaction.

The perturbative analysis of our model shows, in fact, the existence of a free fixed-point that is stable in the infrared limit. This can be understood from the nontrivial scaling of the model with respect to variations of the bandwidth cutoff E_c, that is needed to regulate the divergent contribution of virtual processes to observable quantities. According to the RG point of view, a reduction of the cutoff E_c implements a partial integration of the high-energy electron modes, that renormalize in this way the value of the effective parameters in the low-energy theory. The electron charge e is not renormalized in the present model, but the Fermi velocity v_F is renormalized to first order in perturbation theory by a self-energy correction of the form[14]

$$\Sigma(\mathbf{k},\omega_k) \approx \frac{e^2}{8\pi}\boldsymbol{\sigma}\cdot\boldsymbol{k}\log E_c \tag{4}$$

Thus, in the perturbative regime the Fermi velocity v_F grows steadily as the cutoff E_c is reduced, and the effective coupling constant e^2/v_F flows to zero in the low-energy effective theory.

The most interesting point, however, concerns the analysis of the model away from the perturbative regime. With regard to the direct application to the graphite system the weak coupling results are of little use, since the bare coupling in the graphite sheet has an estimated value $e^2/v_F \sim 10$. Some possibly relevant effects like, for instance, the renormalization of the quasiparticle weight have to be consistently understood in a nonperturbative framework.

As stated above, the polarization tensor does not show any divergence with respect to the bandwidth cutoff E_c and, at the one-loop level, it is given by

$$i\Pi(\mathbf{k},\omega_k) = i\frac{e^2}{8}\frac{\mathbf{k}^2}{\sqrt{v_F^2\mathbf{k}^2-\omega_k^2}} \tag{5}$$

The dressed propagator of the interaction in the RPA is given by

$$\langle\phi(\mathbf{k},\omega_k)\phi(-\mathbf{k},-\omega_k)\rangle = \frac{-i}{2|\mathbf{k}| + \frac{e^2}{8}\frac{\mathbf{k}^2}{\sqrt{v_F^2\mathbf{k}^2-\omega_k^2}}} \tag{6}$$

By using the dressed propagator (6) in the computation of the self-energy one is able to perform a partial sum of perturbation theory, in which one takes into account the set of most singular diagrams with regard to the bare interaction $\sim 1/|\mathbf{k}|$. In this kind of GW approximation and for the model with conical dispersion relation, we have

$$i\Sigma(\mathbf{k},\omega_k) = i2e^2 \int \frac{d^2p}{(2\pi)^2}\frac{d\omega}{2\pi}\;\frac{\omega_k-\omega+v_F\boldsymbol{\sigma}\cdot(\mathbf{k}-\mathbf{p})}{v_F^2(\mathbf{k}-\mathbf{p})^2-(\omega_k-\omega)^2}\;\frac{-i}{2|\mathbf{p}| + \frac{e^2}{8}\frac{\mathbf{p}^2}{\sqrt{v_F^2\mathbf{p}^2-\omega^2}}} \tag{7}$$

The imaginary part of the self-energy coming from (7) has been computed elsewhere[17], and it has been shown to have a linear dependence on quasiparticle energy, consistent with the measured quasiparticle lifetimes in graphite. In

this paper we are interested in the computation of the real part of $\Sigma(\mathbf{k}, \omega_k)$, which provides information about the nontrivial scaling of the quasiparticle weight and the Fermi velocity in the low-energy limit.

The terms linear in ω_k and $\mathbf{k}$ in the self-energy (7) display a logarithmic dependence on the high-energy cutoff E_c. This can be determined in the following way. Since we are interested in the real part of Σ we may perform the analytic continuation $\omega \to i\,\overline{\omega}$. We end up with the expression:

$$
\begin{aligned}
i\Sigma(\mathbf{k}, \omega_k) = & \; i\frac{e^2}{v_F} \int \frac{d^2p}{(2\pi)^2} \frac{d\overline{\omega}}{2\pi} \frac{1}{|\mathbf{p}|} \frac{\overline{\omega}_k - \overline{\omega} + \boldsymbol{\sigma}\cdot(v_F \mathbf{k} - \mathbf{p})}{(v_F \mathbf{k} - \mathbf{p})^2 + (\overline{\omega}_k - \overline{\omega})^2} \\
& - i\frac{e^2}{v_F} \frac{e^2}{16 v_F} \int \frac{d^2p}{(2\pi)^2} \frac{d\overline{\omega}}{2\pi} \frac{\sqrt{\mathbf{p}^2 + \overline{\omega}^2} - \frac{e^2}{16 v_F}|\mathbf{p}|}{\alpha \mathbf{p}^2 + \overline{\omega}^2} \frac{\overline{\omega}_k - \overline{\omega} + \boldsymbol{\sigma}\cdot(v_F \mathbf{k} - \mathbf{p})}{(v_F \mathbf{k} - \mathbf{p})^2 + (\overline{\omega}_k - \overline{\omega})^2}
\end{aligned}
\tag{8}
$$

where $\alpha \equiv 1 - (e^2/(16 v_F))^2$. A singularity at $\alpha = 0$ appears in the above expression, whose role has to be clarified since there is no sign of a particular feature at $e^2/(16 v_F) = 1$ in the original expression. We will perform the above integrals taking the value of $\alpha > 0$, but it will become clear at the end that the results can be continued smoothly to the strong coupling regime $e^2/(16 v_F) > 1$.

Upon integration of the frequency from $-\infty$ to $+\infty$ and placing the bandwidth cutoff E_c in momentum space, $v_F |\mathbf{p}| < E_c$, the coefficients of the logarithmically divergent contributions can be computed in terms of elementary functions of $g \equiv e^2/(16 v_F)$. The renormalization of the electron propagator turns out to be given by

$$
\begin{aligned}
\frac{1}{G} &= \frac{1}{G_0} - \Sigma \\
&= Z^{-1}(\omega_k - v_F \boldsymbol{\sigma}\cdot\mathbf{k}) \\
& \quad - Z^{-1}\omega_k \frac{8}{\pi^2}\left(g^2 + \left(2 - g^2\right)\left(1 - \frac{\arcsin g}{g\sqrt{1 - g^2}}\right)\right) \log E_c - Z^{-1}\omega_k \frac{8}{\pi}\frac{1}{g}\left(\frac{1 - g^2/2}{\sqrt{1 - g^2}} - 1\right) \log E_c \\
& \quad + Z^{-1} v_F \boldsymbol{\sigma}\cdot\mathbf{k} \frac{8}{\pi^2}\left(1 - \frac{\sqrt{1 - g^2}}{g} \arcsin g\right) \log E_c - Z^{-1} v_F \boldsymbol{\sigma}\cdot\mathbf{k} \frac{4}{\pi}\frac{1}{g}\left(1 - \sqrt{1 - g^2}\right) \log E_c
\end{aligned}
\tag{9}
$$

where $Z^{1/2}$ represents the scale of the bare electron field compared to that of the cutoff-independent electron field

$$
\Psi_{bare}(E_c) = Z^{1/2}\Psi \tag{10}
$$

In the RG approach, we require the cutoff-independence of the renormalized Green function, since this object leads to observable quantities in the quantum theory. For this purpose, Z and v_F have to be understood as cutoff-dependent effective parameters, that reflect the behavior of the quantum theory as $E_c \to 0$ and more states are integrated out from high-energy shells of the band. We get the RG flow equations

$$
E_c \frac{d}{dE_c} \log Z(E_c) = -\frac{8}{\pi^2}\left(2 + \frac{2 - g^2}{g}\frac{\arccos g}{\sqrt{1 - g^2}}\right) + \frac{8}{\pi}\frac{1}{g} \tag{11}
$$

$$
E_c \frac{d}{dE_c} v_F(E_c) = -\frac{8}{\pi^2} v_F \left(1 + \frac{\arccos g}{g\sqrt{1 - g^2}}\right) + \frac{4}{\pi} v_F \frac{1}{g} \tag{12}
$$

Given that the electron charge e is not renormalized, we may write down the flow equation for the effective coupling constant $g = e^2/(16 v_F)$

$$
E_c \frac{d}{dE_c} g(E_c) = \frac{8}{\pi^2}\left(g + \frac{\arccos g}{\sqrt{1 - g^2}}\right) - \frac{4}{\pi} \tag{13}
$$

In the weak coupling regime, one may check that the renormalization of both the Fermi velocity and the electron wavefunction takes place in the expected direction. The quasiparticle weight Z at small g is smaller than the bare value measured before integration of high-energy modes. The Fermi velocity v_F flows to higher values in the infrared, and the density of states around the Fermi energy decreases, as a consequence of screening effects. This ensures the consistency of the weak coupling phase, where the results of perturbation theory become increasingly reliable in the low-energy limit. However, the most important point concerns the possible existence of a different phase at large values of g. In this respect, the flow equations (11) and (13) can be analytically continued to values $g > 1$, by simple

use of the formula $\arccos z = i \log(z - i\sqrt{1-z^2})$. The flows of the coupling constant and the electron wavefunction are then differentiable across $g = 1$, which shows that the apparent singularity at this point has no physical meaning.

The right-hand-side of Eq. (13) is a monotonous function of g, taking into account the mentioned analytic continuation. This means that there is no phase different from that of the perturbative regime, and that the strong coupling regime is connected to it through RG transformations. The RG flow represented in Fig. 1 shows that the perturbative regime is attained at low-energies, starting from fairly large bare values of the coupling constant.

The present analysis is relevant to the phenomenology of the graphite layers. It shows that, even in such a system with unconventional quasiparticle lifetimes, the low-energy behavior is governed by a fixed-point which can be described as a Fermi liquid, as Z tends to a constant in the infrared, unlike for the line of nontrivial fixed-points which characterize Luttinger liquids. The low-energy scaling of the quasiparticle weight is represented in Fig. 2. Hence we can assert that, although the quasiparticles decay according to the experiment with a rate proportional to their energy, rather than to their energy square, the notion of well-defined quasiparticles with a finite weight at the Fermi level still holds in the system.

Our study stresses the anomalous screening properties of the Coulomb interaction in low-dimensional systems. This fact has been also put forward recently in a different framework, pointing out that in one and two-dimensional systems the screening of the long-range interactions goes in the direction of reducing the electron correlations[20]. In the RG approach we see how this effect arises under the form of a renormalization of v_F. It is remarkable that such nontrivial scaling of the Fermi velocity in the infrared operates in the two-dimensional as well as in the one-dimensional system with Coulomb interaction[21], and it seems to be also present in systems with magnetic interactions[12,13].

The analysis presented here is relevant to the controversy about the existence of systems with Fermi liquid behavior in two dimensions. Our approach is completely rigorous in the limit of a very large number N of electron flavors, where the bubble summation implicit in the RPA becomes exact. The leading order in the $1/N$ expansion provides, on the other hand, the fixed-point and the exact anomalous dimension of the electron field in the Luttinger model[19,21]. Therefore, this approximation should suffice to describe such kind of nontrivial fixed-point, in case that it were present in the model.

To summarize, semimetals described by the two-dimensional Dirac equation, such as a graphite layer, show significant differences with respect to the properties of standard Fermi liquids with (screened) Coulomb interactions. In the present problem, the quasiparticle lifetime goes like $\sim \omega^{-1}$, while the enhancement of the Fermi velocity implies the vanishing of the effective coupling in the infrared. We expect the same behavior in three-dimensional zero-gap semiconductors, which are also described by an effective Dirac equation.

In the context of more general long-range interactions, the system with a Dirac sea behaves also differently with respect to the conventional Fermi sea, as in the former an interaction $V(q) \sim 1/|q|^{1+\epsilon}$ already departs from the Fermi liquid universality class for $\epsilon > 0$. In our RG framework the interaction becomes relevant no matter how small ϵ may be, and a nontrivial fixed-point can be found away from the origin within the ϵ expansion.

The logarithmic behavior that we have found affects also some thermodynamic quantities like the specific heat or the susceptibility, which pick up logarithmic corrections as $T \to 0$. These are the signature of the marginal character of the interaction, which has in our model the precise degree of singularity to separate regimes with Fermi liquid ($\epsilon < 0$) and non-Fermi liquid behavior ($\epsilon > 0$).

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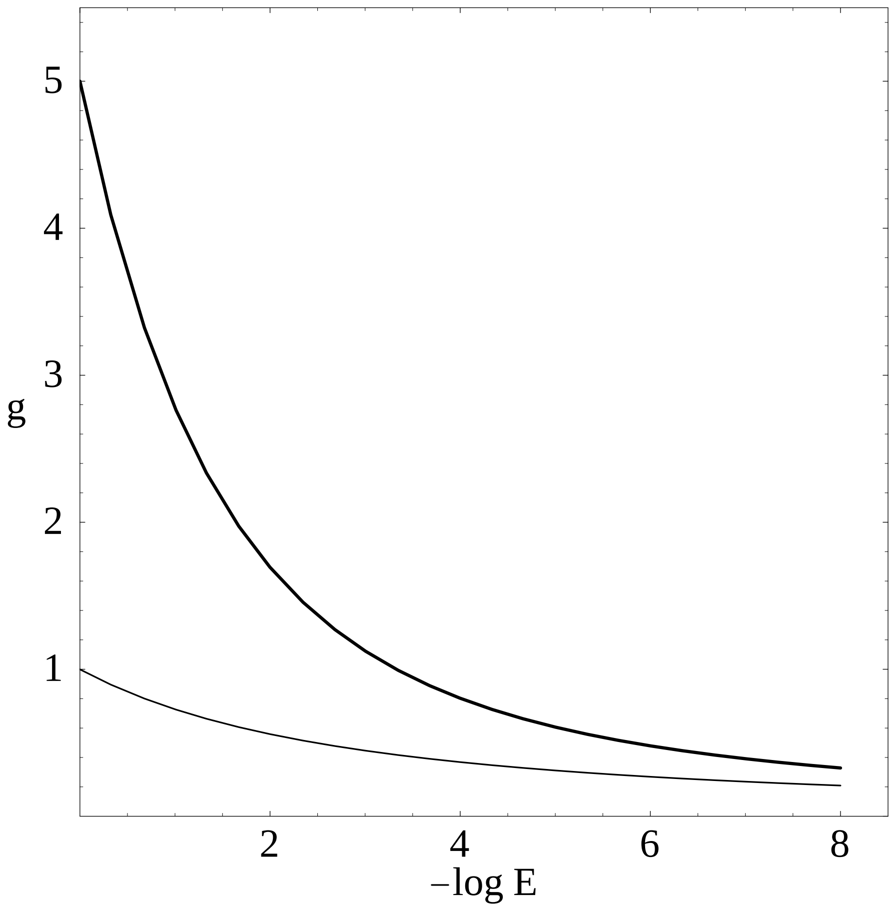
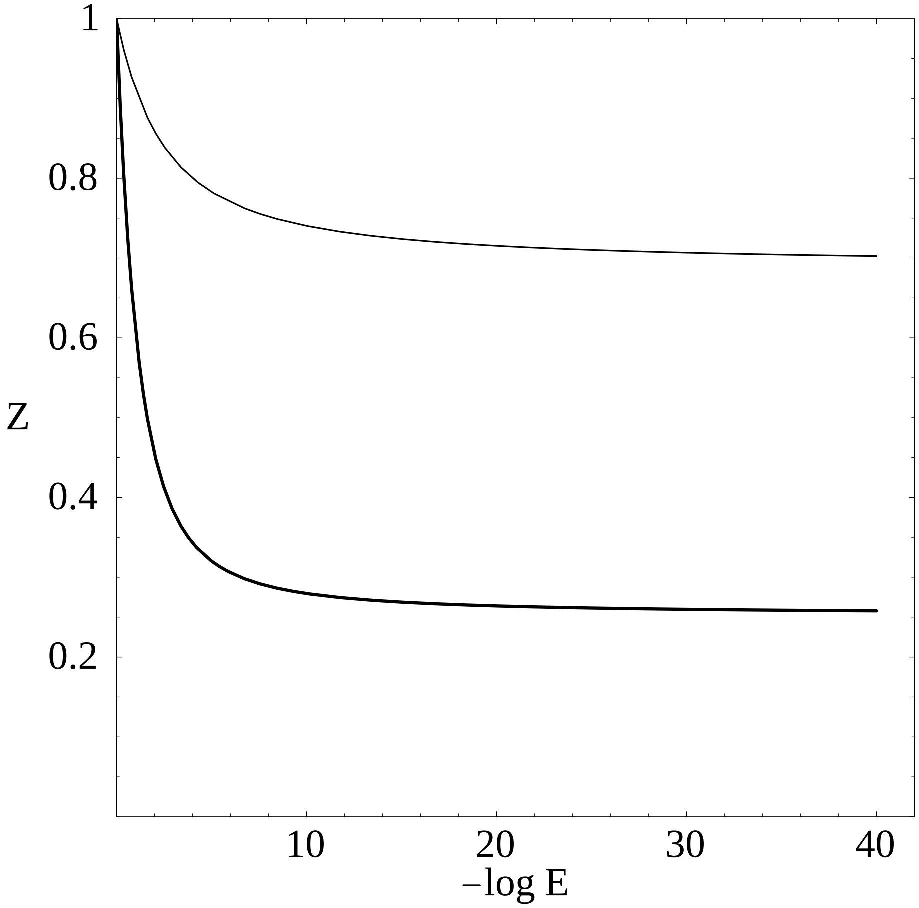
FIG. 1. Flow of the coupling constant for different bare values.

FIG. 2. Wavefunction renormalization for bare coupling constant $g = 5$ (thick line) and $g = 1$ (thin line).



5
4
3
2
1
g
2
4
6
8
−log E



1
0.8
0.6
Z
0.4
0.2
10
20
30
40
−log E